Performance Analysis of BPSK and QPSK Based on Bit Error Rate

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Abstract

This paper presents a comparison of BPSK & QPSK modulation techniques based on BER measurements. The SNR v/s BER tabulation & curves for the above modulation techniques are presented using MATLAB/Simulink model. The result shows that the BPSK gives negligible BER after SNR > 10dB but there is a significant BER in QPSK compare to BPSK.

Keywords: Bit Error Rate (BER), Signal-to-Noise Ratio (SNR), Binary Phase Shift Keying (BPSK), Quadrature Phase Shift Keying (QPSK).

1. Introduction

The selection of the particular modulation technique used is determined by the application intended as well by the channel characteristics such as available bandwidth and the susceptibility of the channel to fading. A fading channel is one in which the received signal amplitude varies with time because of variability in the transmission medium. In each of these situations we need a modulator at the transmitter end, at the receiver, a demodulator to recover the baseband signal. Such a modulator-demodulator combination is called MODEM. In this paper we present a description of BPSK & QPSK modulation technique and compare them on the basis of their Bit Error Rate.

In [1] BER analysis of BPSK, simulation result offers approximately 40-50% improvement over the measurement results. In [2] highlights the performance analysis of BPSK and QPSK using error correcting code. The performance was determined in term of bit rate error (BER) and signal energy to noise power density ratio (Eb/No). In [3] the performance of OFDM-BPSK and-QPSK system in Nakagami-m channel has been reported.

2. Binary Phase Shift Keying

2.1 Spectrum of BPSK

The baseband signal \( b(t) \) is a NRZ (non return to zero) binary waveform whose power spectral density is given by

\[
G_b(f) = P_e T_b \left( \frac{\sin \pi f T_b}{\pi f T_b} \right)^2
\]

(1)

The BPSK waveform is the NRZ waveform multiplied by \( \sqrt{2} \cos \omega_c t \). The power spectral density of the BPSK signal is

\[
G_{BPSK} = \frac{P_e T_b}{2} \left[ \frac{\sin \pi f f_0 T_b}{\pi f f_0 T_b} \right]^2
\]

(2)

In principle at least, the spectrum of \( G_b(f) \) extend over all frequencies and correspondingly so does \( G_{BPSK}(f) \). Suppose then we tried to multiplex signals using BPSK, using different carrier frequencies for different baseband signals. There would inevitably be overlap in the spectra of the various signals and correspondingly a receiver tuned to one carrier would also receive, albeit at a lower level, a signal in different channel. This overlapping of spectra causes inter-channel interference.

2.2 Geometrically Representation of BPSK Signal

BPSK signal can be represented in terms of an orthogonal signal.
\[ u_1(t) = \sqrt{2/T_b} \cos \omega_0 t \]

\[ v_{BPSK}(t) = \left[ \sqrt{P_s T_b} b(t) \right] \sqrt{2/T_b} \cos \omega_0 t \]

\[ = \left[ \sqrt{P_s T_b} b(t) \right] u_1(t) \quad (3) \]

The distance between signals is

\[ d = 2 \sqrt{P_s T_b} = 2 \sqrt{E_b} \quad (4) \]

Where \( E_b = P_s T_b \) is the energy contained in the bit duration. The distance \( d \) is inversely proportional to the probability that we make an error when, in the presence of noise, we try to determine which of the levels of \( b(t) \) is being received.

3. Quadrature Phase Shift Keying

We have seen that when a data stream whose bit duration is \( T_b \) is to be transmitted by BPSK the channel bandwidth must be nominally \( 2f_s \) where \( f_s = 1/T_b \).

Quadrature Phase Shift Keying, as well as explain, allows bit to be transmitted using half the bandwidth.

Symbol v/s Bit Transmission

In BPSK we deal individually with each bit of duration \( T_b \) in QPSK we lump two bits together to form what is termed as symbol. The symbol can have any one of four possible values corresponding to the two-bit sequences 00, 01, 10 and 11. We therefore arrange to make available for transmission four distinct signals. At the receiver each signal represents one symbol and, correspondingly, two bits. When bits are transmitted, as in BPSK, the signal changes occur at the bit rate. When symbols are transmitted the changes occur at the symbol rate which is one-half the bit rate. Thus symbol time is \( T_s = 2T_b \).

3.1 Signal Representations

In section (3.5.2) we investigated four Quadrature signals Eq. (3.5.1) can be rewritten as:

\[ v_m(t) = \sqrt{2P_s} \cos \left[ \omega_0 t - (2m - 1)\frac{\pi}{4} \right] \quad m = 0, 1, 2, 3 \quad (5) \]

These signal were then represented in terms of the two orthogonal signals

\[ u_1 = \sqrt{2/T} \cos \omega_0 \]

and \[ u_2 = \sqrt{2/T} \sin \omega_0 t. \]

The result in Eq. (3.5.2) repeated here,

\[ i \nu_m(t) = \left[ \sqrt{P_s T} \cos (2m - 1) \frac{\pi}{4} \right] \frac{2}{\sqrt{T}} \cos \omega_0 t \]

\[ - \left[ \sqrt{P_s T} \sin (2m - 1) \frac{\pi}{4} \right] \frac{2}{\sqrt{T}} \sin \omega_0 t \quad (6) \]

The QPSK signal \( v_m(t) \) in Eq. (3.5.1) can be put in the form of Eq. (3.5.3) by setting

\[ b_e = \sqrt{2} \cos (2m - 1) \frac{\pi}{4} \quad (7) \]

and

\[ b_o = - \sqrt{2} \sin (2m - 1) \frac{\pi}{4} \quad (8) \]

Thus

\[ v_m(t) = \sqrt{E_b} b_e(t) u_1(t) - \sqrt{E_b} b_o(t) u_2(t) \quad (9) \]

Where \( T = 2T_b = T_s \).

The point in signal space corresponding to each of the four possible transmitted signals is indicated by dots.

From each such signal we can recover two bits rather than one. The distance of a signal point from the origin is \( \sqrt{E_s} \) which is the square root of the signal energy associated with the symbol, that is \( E_s = P_s T_s = P_s (2T_b) \).

\[ \sqrt{E_s} = \sqrt{2P_s T_b} = \sqrt{P_s T_s} \]

Fig. 1: The four QPSK signal drawn in signal space
As we have noted earlier our ability to determine a bit without error is measured by the distance in signal space between points corresponding to the different value of the bit. As shown in fig. (1) points which differ in a single bit are separated by the distance

\[ d = 2\sqrt{P_x T_b} = 2\sqrt{E_b} \]  \hspace{1cm} (10)

Where \( E_b \) is energy contained in a bit transmitted for a time \( T_b \). This distance for QPSK is the same as for BPSK. Hence altogether we have the important result that, in spite of reduction by a factor of two in the bandwidth required by QPSK in comparison with BPSK, the noise immunity of the two systems are same.

4. Error Probability

4.1 Error Probability for BPSK

Using the consideration of previously we shall calculate the error probability for synchronous detection in the case of BPSK. The synchronous detector for BPSK is shown in figure (2). Since the BPSK signal is one dimensional, the result that the only relevant noise in the present case is:

\[ n(t) = n_\eta u(t) = n_\eta \sqrt{\frac{2}{T_b}} \cos \omega_0 t \]  \hspace{1cm} (11)

in which \( n_\eta \) is a Gaussian random variable of variance \( \sigma_\eta^2 = \eta/2 \). Now let us suppose that \( s_j \) was transmitted. The error probability, i.e. the probability that the signal is mistakenly judged to be \( s_j \) is a probability that \( n_\eta > \sqrt{P_x T_b} \). Thus the error probability \( P_e \) is:

\[ P_e = \frac{1}{2\sqrt{2\pi}\eta} \int_{-\infty}^{\infty} e^{-n_\eta^2/2\eta} \, dn_\eta \]

\[ = \frac{1}{\sqrt{\pi\eta}} \int_{-\infty}^{\infty} e^{-y^2} \, dy \]

\[ = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{P_x T_b}{\eta}} \right) \]  \hspace{1cm} (13)

The signal energy is \( E_b = P_x T_b \) and the distance between end points of the signal vectors in fig. (3) is \( = 2\sqrt{P_x T_b} \). Accordingly we find that

\[ P_e = \frac{1}{2} \text{erfc} \left( \frac{E_b}{\sqrt{\eta}} \right) = \frac{1}{2} \text{erfc} \left( \frac{d^2}{4\eta} \right) \]  \hspace{1cm} (14)

The error probability is thus seen to fall off monotonically with an increase in distance between signals.

4.2 Error Probability for QPSK

In QPSK one of four possible waveforms is transmitted during each interval \( T \). These waveforms are:

\[ S_i(t) = A \cos \left( \omega_0 t + [2m - 1] \frac{\pi}{4} \right) \]  \hspace{1cm} (15)

\( m = 1, 2, 3, 4 \)  \hspace{1cm} \( 0 \leq t \leq T_s = 2T \)

These four waveforms are represented in the phasor diagram of fig. (3).

Fig. 2: Correlator receiver for BPSK showing that \( r = r_1 + n_1, or r_2 + n_2 \)

Fig. 3: A phasor representation of the signal in QPSK

The receiver system is shown in fig. (3). Observe that two correlators are required and that the local reference waveforms, as indicated also in fig. (4.5.1), are \( A \cos \omega_0 t \) and \( A \sin \omega_0 t \).

The probability of error of the system is

\[ P_e(QPSK) = 1 - P_e \leq 2P_e = \text{erfc} \left( \frac{d^2}{4\eta} \right) \]  \hspace{1cm} (16)

5. SIMULINK Model

In this Simulink model I have design different digital modulation techniques in binary phase and measure bit error rate for different value of SNR, and plot SNR/BER graph for different techniques.
Table: 1 SNR v/s BER for BPSK & QPSK

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<th>SNR</th>
<th>BPSK</th>
<th>QPSK</th>
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Figure: 8 Plot for BER v/s SNR for BPSK & QPSK

6. Conclusion

In this paper, a comparison of BER between BPSK & QPSK techniques for different modulation SNR value is calculated and shown graphically by Simulink model. By analyzing graphically representation for SNR ranging within 0≤SNR≤13, there is a significant decrease in BER for BPSK is obtained. At SNR>10, BER for BPSK shift to 0. These calculation conclude that the BPSK modulation techniques is better as compared to QPSK as far as BER is concerned.
7. References


